

Thermodynamic Compressibility of a Two-Dimensional Electron System: Signature of a Droplet State

S. C. Dultz, B. Alavi, and H. W. Jiang

Department of Physics and Astronomy, University of California at Los Angeles, Los Angeles, CA 90095
(February 1, 2008)

We have used a field-penetration method to measure thermodynamic compressibility of a moderately interacting two-dimensional electron system ($r_s \approx 0.5-3$) in a three terminal GaAs/AlGaAs device, fabricated with an epitaxial lift-off technique. We found that the density and temperature dependencies of the compressibility are qualitatively different from that observed in earlier studies of the 2D hole system, where interaction energies are considerably stronger. We show that the observed characteristics can be described by the recently developed formalism for compressibility of the droplet state.

The apparent metal-insulator transition discovered in low-density, high mobility Si, SiGe and GaAs based two-dimensional semiconductor systems continues to capture much interest in the general condensed matter physics community [1]. Although the experimental work in this research area has been predominately transport studies, there is recent work, both experimental [2–4] and theoretical [5,6] devoted to the study of the thermodynamic compressibility of the interaction of disordered two-dimensional systems, in the context of the 2D metal-insulator transition.

In our earlier measurement of a strongly correlated two-dimensional hole system (2DHS) in a p-type GaAs/AlGaAs heterostructure, we found that the negative inverse-compressibility at low-densities reaches a minimum value at the apparent metal-insulator transition and then increases dramatically with further decreasing density. The results show a strong correlation between the qualitative change in thermodynamic compressibility and the transport signature of the 2D metal-insulator transition. In that experiment, the 2DHS has a rather strong correlation due to the heavy effective mass. The dimensionless parameter r_s which measures the ratio of the correlation energy to the Fermi energy ranges between 5 to 25 and the qualitative change in the compressibility was observed at $r_s=13$. To understand whether this behavior is specific to strongly correlated systems and how it is linked to the 2D metal-insulator transition, we have studied another two-dimensional electron system (2DES) where interactions are much weaker, $r_s = 0.5-3$. There is no transition in the transport. We found that the compressibility behavior in the present system is qualitatively different from that observed in the 2DHS system. Its density dependence can be well described by the theory [6] of percolating droplet states.

To measure the thermodynamic compressibility, we have used a field penetration technique, developed initially by Eisenstein. [8] Unique to our setup, the 2DES is sandwiched between two metal electrodes (the front and back gate) as shown in the inset of Fig. 1. By using an epitaxial lift-off technique for removing the MBE substrate, we were able to place a back-gate at a distance of 1000 nm from the 2DES, while the front gate is 167 nm away. The samples used in this experiment were cut

from an n-type GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ wafer fabricated by molecular beam epitaxy. A 70 Å $\text{Al}_{0.30}\text{Ga}_{0.70}\text{As}$ undoped spacer was used with an atomic planar doping layer of Si just above with a concentration of $1.5 \times 10^{12} \text{ cm}^{-2}$. The rest of the sample, designed specifically to do the substrate removal, was identical to the one used for the 2DHS compressibility study [2]. The mobility of the sample was $186,000 \text{ cm}^2/\text{Vs}$ at a density of about $n = 6 \times 10^{11} \text{ cm}^{-2}$ when the sample was unbiased. The wafer was carefully designed so that the mobility of the carriers was comparable to that in the 2DHS studied earlier.

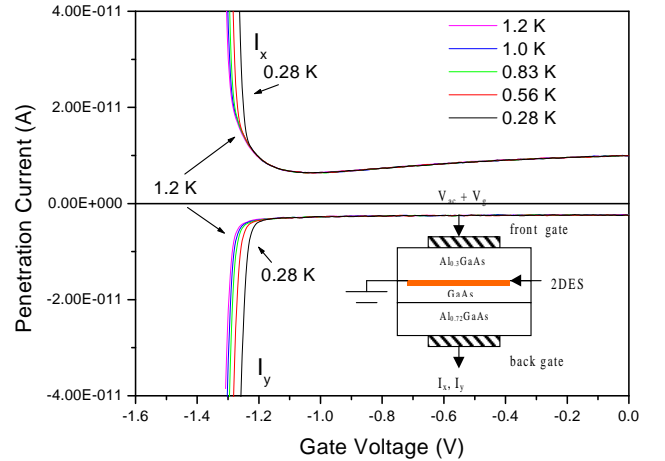


FIG. 1. Penetration current components I_x and I_y vs gate voltage for five temperatures at an excitation frequency of 100 Hz: the temperatures are labeled from right to left as 0.28 K, 0.56 K, 0.83 K, 1.0 K and 1.2 K respectively. Inset: the schematic of the experimental setup.

To perform the experiment, we applied a 10 mV AC excitation voltage V_{ac} to the front gate. A DC voltage V_g was superimposed to control the carrier density. With the 2DES grounded, the penetration current from the front gate to the back gate was detected by a lock-in amplifier at an excitation frequency of 100 Hz. The frequency was chosen in such a way as to maximize our signal-to-noise ratio while remaining in a frequency independent regime.

As described before [2], by modeling the system as a

distributed circuit, both the quantum capacitance C_q and the resistance R_s of the channel for the 2D carriers could be extracted individually based on the measured values for the in-phase I_x and 90° phase I_y current components:

$$I = \frac{i\omega C_1 C_2 V_{ac}}{C_1 + C_2} \left[1 - \left(\frac{C_q}{C_1 + C_2 + C_q} \right) \frac{\tanh(\alpha)}{\alpha} \right], \quad (1)$$

$$\alpha = \sqrt{i\omega \frac{C_q(C_1 + C_2)}{C_1 + C_2 + C_q} R_s}. \quad (2)$$

where ω is the frequency of the excitation voltage, and C_1 and C_2 are the geometric capacitances between the front and back gates with respect to the 2D layer.

The quantum capacitance per unit area c_q is related to the compressibility, $\kappa = (dn/d\mu)/n^2$, by $c_q = e^2 (dn/d\mu)$, where μ is the chemical potential and n is the carrier density.

Figure 1 shows both the in-phase I_x and 90° phase I_y components of the penetrating current as a function of the gate voltage for five different temperatures ranging from 0.28 K to 1.2 K. The $d\mu/dn$ value, which is inversely proportional to the compressibility, is extracted and plotted in Figure 2 as a function of the r_s value in the bottom axis. At 100 Hz, $d\mu/dn$ is almost directly proportional to I_x . There are important differences between this curve and the same curve for the 2DHS. First, it is immediately apparent from the data in Figure 1 that the resistive component of the signal does not have a temperature independent crossing point like that for the 2DHS. This should be expected from a sample with only weak to strong localization crossover behavior as shown in the transport measurements. The transport properties of the 2DES can be understood in terms of the more conventional scaling theory. [7] Secondly, $d\mu/dn$ for the 2DES gas is positive everywhere. At high densities, this dependence is expected. As one can see from the quantitative agreement with the Hartree-Fock (HF) approximation calculation for low r_s values. [9]. Since the effective mass of electrons in GaAs is only 0.067 times the rest mass of an electron (roughly 5 times less than the effective mass of the holes in GaAs), the maximum r_s value of this particular system is only about 3 for the lowest density studied here. Positive compressibility means that the kinetic energy is always greater than the exchange energy for the present system. Third, although there is still a minimum in $d\mu/dn$, just like that observed in the 2DHS, the turn-around is not nearly as sharp as is the case for that system and occurs at a density of roughly ($r_s = 1.4$). Finally, the temperature dependence of $d\mu/dn$ is rather strong, in contrast to what is observed in the 2DHS where $d\mu/dn$ is temperature independent for a large range of temperatures from 0.3 K to 4.2 K.

Over the years, it has been suggested that the interacting 2D electrons in the presence of disorder can be understood by the droplet, or puddle, model. [11,12] In this model, the electrons are separated into “liquid” droplets with local density higher than the average density and

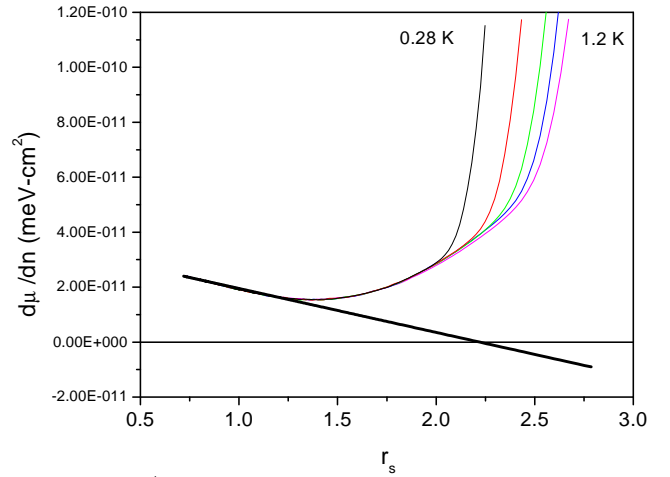


FIG. 2. $d\mu/dn$ is plotted against r_s for the data in Figure 1. The solid line is calculated using HF approximation.

“gas” islands with lower density. For a given disorder, due to the potential fluctuation of the donors, as the Fermi energy is decreased, the “gas” region increases. In this model, a cross-over from metallic behavior to insulating behavior should be seen around the percolation threshold. [13]

Specifically, the thermodynamic compressibility has been recently calculated by Shi and Xie for an interacting system within the local density approximation (LDA). [6] The LDA result produced a “turn-around” in $d\mu/dn$ which appears to be similar to that observed in the 2DHS system. [2] Furthermore, the LDA calculation was in good agreement with numerical simulations using “liquid” droplets and “gas” islands.

Following the work of Shi and Xie, we have calculated the compressibility numerically using LDA for the experimental parameters and found that the basic characteristics of the electron data can be well described by the formalism of the LDA.

In the LDA calculation, the local density (n_{eff}) rather than the average density, is used to determine the total energy of the system. The local density is assumed to be $n_{eff} = n/f(n)$, where $f(n)$ is the fractional area of the high density region. The chemical potential is therefore:

$$\mu(n) = d(\varepsilon_0(n_{eff})n)/dn, \quad (3)$$

where ε_0 is the energy density for the uniform electrons and is given by [9]:

$$\varepsilon = \frac{1}{2}\varepsilon_{Fermi} + \varepsilon_{exchange} = \frac{\pi\hbar^2 n}{2m^*} - \frac{4}{3}\sqrt{\frac{2}{\pi}} \frac{e^2}{4\pi\epsilon} n^{\frac{1}{2}}, \quad (4)$$

where ϵ is the dielectric constant of the host material and m^* is the effective mass of the 2DES.

The chemical potential and $d\mu/dn$ are calculated numerically. As shown in Figure 3, the LDA approximation describes the data rather well in the high temperature

limit. To fit the data we use the following relation for $f(n)$:

$$f(n) = \frac{1}{(1 + (n_0/n)^\alpha)} \quad (5)$$

$n_0 = 1.6 \times 10^{11} \text{ cm}^{-2}$ and $\alpha = 2.0$ have been used to produce the theoretical curve in the figure. It turns out that the numerical fit is very sensitive to the value of α and n_0 . Small deviations of about 10 percent from $n_0 = 1.6 \times 10^{11} \text{ cm}^{-2}$ or $\alpha = 2.0$ dramatically effect the curve in such a way that we can no longer qualitatively fit the data. Incidentally, the value of α of 2.0 is very close to that of 2.3 used to fit the numerical simulation by Shi and Xie [6] for off-plane, un-correlated charge impurities and this parameter should depend on the form of the disorder potential. We do not understand why the data at lower temperatures or lower densities deviate from the theoretical curve. It is possible that the temperature dependence of the size of the lower density islands is allowing the field to penetrate to the back gate more easily at the lowest temperatures. This could create a temperature dependence in $f(n)$ in equation 5, which could explain a temperature dependent $d\mu/dn$ curve. However, it is unlikely that the temperature dependence would come about from a single temperature dependent parameter in the above equation. [16]

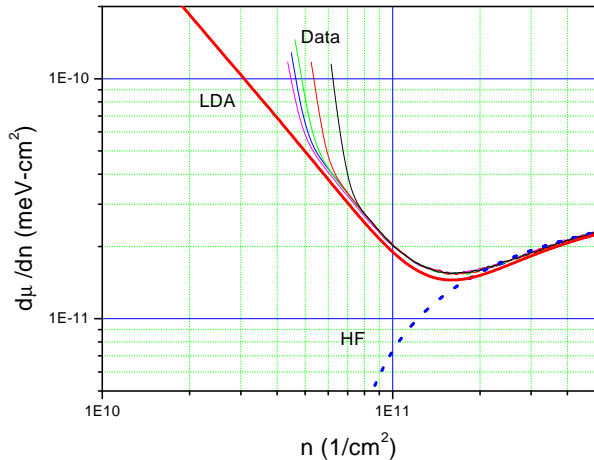


FIG. 3. The data is compared with the LDA approximation. The LDA approximation describes the data well in the high temperature limit. $n_0 = 1.6 \times 10^{11} \text{ cm}^{-2}$ and $\alpha = 2.0$ are used for the fitting.

It is important to note that the accurate data fit in Figure 3 to the theoretical curve should not give the impression that the droplet model explains all experimental data for different systems. In fact, Figure 4 shows that the LDA does not fit the data for the 2DHS density dependence of $d\mu/dp$ at all. To compare with the data in reference [2] we cannot find parameters of α and n_0 which would fit the curve even qualitatively. Incidentally, we can match the data with an expression which contains a

mathematical singularity, which of course is inconsistent with that for percolation. As an example, we plot a curve of the LDA using $f(n) = (1 - (n_0/p)^\beta)$. A reasonable fit is obtained with $n_0 = 3.5 \times 10^{10} \text{ cm}^{-2}$ and $\beta = 0.06$. This observation implies that the physics of the 2DHS which appears in the large r_s regime, CANNOT be described by the droplet model.

The necessity of a singularity in f further suggests that there is a phase transition in the 2DHS system. This observation further supports the notion that the non-analytical behavior of the thermodynamic compressibility is consistent with the unusual signature of the metal-insulator transition observed in the transport for the same sample. Indeed there are theories describing the 2D metal-insulator transition as a phase transition from a liquid to a Wigner glass state [14,15] or from a paramagnetic state to a spontaneously ordered ferromagnetic state [17]. It is difficult to determine the existence of these potential states from these measurements due to the absence of a quantitative analysis of the compressibility of the Wigner glass state or the ordered ferromagnetic state. For example, the compressibility of a Wigner crystal, in the absence of disorder, is known theoretically [18] to be only twice that for a liquid in the HF theory. However, compressibility for a glass state, in the presence of realistic disorder, has not yet been calculated.

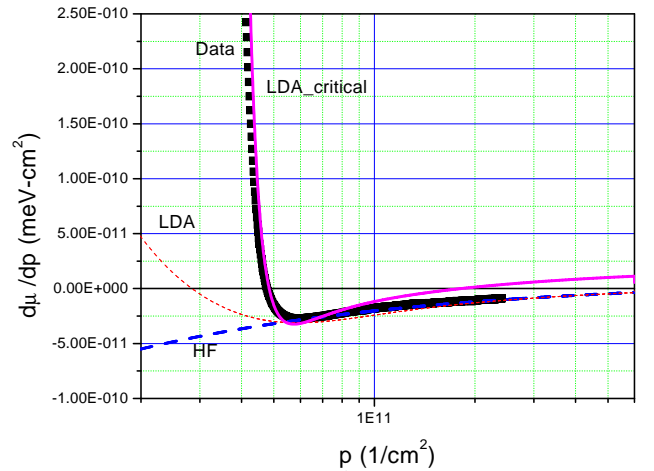


FIG. 4. In contrast, the LDA approximation cannot describe the $d\mu/dp$ vs. p curve obtained in the 2D hole system, since the theoretical curve varies slowly with p . A reasonable fit can only be obtained if there is a singularity in the filling factor $f(n)$, as described in the text.

It is reasonable to speculate about why the droplet model is more appropriate to describe the smaller r_s system. It is now a rather common view that the interaction can suppress the effect of disorder. As shown in the numerical simulation, as r_s becomes more important the system tends to keep the electron density as uniform as possible to minimize the Coulomb interaction. So for the strongly interacting heavy mass hole system, it is possible

that the droplet state is largely suppressed.

In summary, we have measured the compressibility of a moderately interacting 2D electron system ($r_s \approx 0.5-3$). We revealed qualitative differences between this system and that observed earlier in a 2D hole system, where interactions are considerably stronger. We found that the compressibility data for the 2DES can be described in the framework of the droplet state model using the local density approximation.

We would like to thank M. Sarachik and X. C. Xie for useful discussions. This work is supported by NSF under grant # DMR 0071969.

-
- [1] E. Abrahams, S. V. Kravchenko, M. P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001).
 - [2] S. C. Dultz, and H. W. Jiang, Phys. Rev. Lett. **84**, 4689 (2001).
 - [3] S. Ilani, A. Yacoby, D. Mahalu, and H. Shtrikman, Phys. Rev. Lett. **84**, 3133 (2000).
 - [4] S. Ilani, A. Yacoby, D. Mahalu, and H. Shtrikman, Science, **292**, 1354 (2001).
 - [5] Qimiao Si, and C. M. Varma, Phys. Rev. Lett. **81**, 4951 (1998).
 - [6] J. Shi, and X. C. Xie, Phys. Rev. Lett. **88**, 86401 (2002).
 - [7] F. W. Van Keuls, H. Mathur, H. W. Jiang, and A. J. Dahm, Phys. Rev. B **56**, 13263 (1997).
 - [8] J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **50**, 1760 (1994).
 - [9] B. Tanatar and D. M. Ceperley, Phys. Rev. B **39**, 5005 (1989).
 - [10] S. Shapira, U. Sivan, P. M. Solomon, E. Buchstab, M. Tischer, and G. Ben Yoseph, Phys. Rev. Lett. **77**, 3181 (1996).
 - [11] S. He and X. C. Xie, Phys. Rev. Lett. **80**, 3324 (1998).
 - [12] Y. Meir, Phys. Rev. Lett. **83**, 3506 (1999).
 - [13] A. L. Efros, F. G. Pikus, and V. G. Burnett, Phys. Rev. B **47**, 2233 (1993).
 - [14] S. Chakravarty, S. Kivelson, C. Nayak and K. Voelker, Philosophical Magazine B **79**, 859 (1999).
 - [15] A. A. Paster, and V. Dobrosavljevic, Phys. Rev. Lett. **83**, 4642 (2001).
 - [16] private communication with X. C. Xie.
 - [17] C. Chamon, E. Mucciolo, Phys. Rev. Lett. **85**, 5607 (2000).
 - [18] see for example, B.I. Shklovskii, A.L. Efros, JETP Lett. **44**, 123 (1986).